

the inputs to both amplifiers. Because  $|\bar{v}_0^2|$  and  $|\bar{v}_2^2|$  are functions of the equivalent noise resistance of the amplifiers,  $R_n$ , it is desirable to make  $R_n$  as small as possible.  $R_n$  for each amplifier was approximately 770 ohms at 300° K over a band width of 3 to 7 kc/s. This was derived by measuring the recorder deflection (or the squared r.m.s. voltage at the inputs to the multiplier) for  $|\bar{v}_0^2|$  and  $|\bar{v}_2^2|$  for various input resistances to the amplifiers at room temperature. For a band width of 3 to 12 kc/s the equivalent noise resistance of the amplifiers was 650 ohms. If one assumes that the flicker noise is proportional to  $1/f$ , then the equivalent noise resistance of the amplifiers at high frequencies is approximately 340 ohms. Therefore, flicker noise was the main contribution to  $R_n$  between 3 and 7 kc/s.

#### 4. Errors Due to Mismatch of the Time Constants in the $\pi$ Network

Equations (4) and (5) were derived under the condition that  $(\omega\tau_i)^2 \ll 1$  or that all the  $\tau$ 's are equal. If this does not hold, then equation (4) for  $T_0 = T_2$  is modified and one gets:

$$(4a) \quad \frac{T_0(R_0+R_2)}{T_1-2T_0} = R_1 \frac{1+(\omega^2\tau_0\tau_2)/(T_1-2T_0) \{T_1-T_0\tau_1[(1/\tau_0)+(1/\tau_2)]\}}{1+(\omega\tau_1)^2}.$$

If

$$T_1 \gg T_0 \quad \text{and} \quad T_1 \gg T_0\tau_1 \left( \frac{1}{\tau_0} + \frac{1}{\tau_2} \right),$$

one obtains:

$$(4b) \quad \frac{T_0(R_0+R_2)}{T_1-2T_0} \simeq R_1 \frac{1+(\omega\tau_1)^2(\tau_0\tau_2/\tau_1^2)}{1+(\omega\tau_1)^2} = R_1 f(\omega, \tau_i).$$

At helium temperatures it is then sufficient to make  $\tau_0\tau_2 \simeq \tau_1^2$ . The deviation of  $f(\omega, \tau_i)$  from unity will increase with increasing frequency. If both  $\tau_0$  and  $\tau_2$  are 10% larger than  $\tau_1$ , then the error at the upper half power frequency is less than 1%. The average error is smaller, because for lower frequencies the error decreases and  $\tau_1$  was always adjusted between  $\tau_0$  and  $\tau_2$ . For a systematic error in adjusting  $\tau_1$  the fractional error in the noise temperature is almost a constant in the liquid helium range.

#### 5. A-c. Resistance of Thermometer Elements

The deviation of the resistance of  $R_0$ ,  $R_1$ , and  $R_2$  in the audio-frequency range from their d-c. value was estimated to be approximately 0.1% (see above).

#### 6. Response of the Integrator

In the case of a narrow square noise band of width  $B$  and uniform spectral intensity a RC integrator of integration time  $\tau$  will measure with a relative error of a single measurement,  $\beta$ , (Burgess 1951):

$$(8) \quad \beta = \frac{[(A-\bar{A})^2]^{\frac{1}{2}}}{\bar{A}} = (2B\tau)^{-\frac{1}{2}},$$

where  $\bar{A}$  is the deflection of the recorder due to the d-c. component of the signal and  $[(A-\bar{A})^2]^{\frac{1}{2}}$  the mean-square deviation of the recorder due to the signal.

For  $B = 4$  kc/sec and  $\beta = 5 \times 10^{-3}$  (0.5%)  $\tau$  should be at least 5 seconds. Integration times from 22.5 to 90 seconds were used. The band widths of both amplifiers were approximately equal.

#### 7. Lead Corrections

If one assumes that the temperature of the leads going to  $R_0$  and  $R_2$  are at room temperature (worst possible case), then the lead resistance should be less than 0.3 ohm for errors smaller than 0.5%, a requirement not difficult to satisfy.

#### 8. Pickup

To avoid errors due to 60 c.p.s. pickup the lower half power points of the amplifiers were designed at approximately 3 kc/sec. Because no shielded room was available experiments could be performed only at night with fluorescent light, thyatron rectifiers, d-c. motors, etc. turned off. Although the amplifiers were protected against shock, audio noise was easily picked up. The voltages were constantly monitored oscillographically at the inputs of the multiplier, to check the randomness of the noise.

### IV. EXPERIMENTAL RESULTS AND DISCUSSION

Table I shows the boiling points of liquid oxygen and liquid nitrogen measured with approximately 4.48 k $\Omega$  metal film deposit resistors ( $R_0$  and  $R_2$ ) at barometric pressure with an integration time of 90 seconds. They were

TABLE I

Temperatures derived from the vapor pressure,  $T$ , measured noise temperatures,  $T_0$ , and their ratios for the boiling points of oxygen and nitrogen at barometric pressure

$T$ , °K	$T_0$ , °K(meas.)	$T_0/T$
90.23	90.26 ± 0.06	1.000 ± 0.001
77.33	77.25 ± 0.08	0.999 ± 0.001

found to be within 0.2% of the temperatures determined from the vapor pressure. The results of the noise-temperature measurements at helium temperatures are shown in Fig. 3. Because, as pointed out in the introduction, the noise power of the real part of an impedance is a universal function of frequency and temperature, any systematic deviation of the noise-temperature can only be due to experimental error of the equipment. The plot in Fig. 3(a) can be fitted best by an equation of the form:

$$(9) \quad \frac{T_0}{T} = \frac{A}{T} + b,$$

where  $A = 0.385^\circ$  K and  $b = 1$  for this thermometer. The term  $A/T$  can be explained due to errors which can be represented by noise-current sources in shunt with the  $\pi$  network. Equation (7) shows that this error must be proportional to  $1/T$  and the constant of proportionality is  $[e(I_1 + I_2 + I_3) + 2k\alpha T_1/R_g]R_0/2k$ . If errors due to  $R_g$  are neglected, then in the above experi-